CHAPTER THREE
TRANSFORMATION

Introduction:

There are various types of transformation and the types to be considered are:

1. Translation . 3. Rotation.
2. Reflection. 4. Enlargement.

Translation:

- This is the types of transformation in which every point moves the same distance, and in the same direction.
- Under translation, the lengths of lines and the sizes of angles do not change
- This implies that if a figure undergoes translation, its size as well as its angles remain unchanged.
- If the point (x, y) is translated by the vector $\binom{a}{b}$, then

$(x, y) \longrightarrow (x + a, y + b)$,

ie (x, y) tranlation by vector$\binom{a}{b}$ $\longrightarrow$ $(x + a, y + b)$.

Example (1)

If $(x, y)$is translated by the vector $\binom{1}{4}$, then

$(x, y) \longrightarrow (x + 1, y + 4)$.

Example (2)

If $(2, 5)$ is translated by the vector $\binom{1}{3}$, then

$(2, 5) \longrightarrow (2 + 1,5 + 3)$.

$(2,5) \longrightarrow (3,8)$.

N/B: The point (3, 8) is called the image of the point (2, 5).

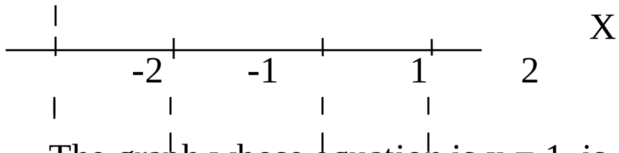
Reflection :

The reflection of a point or a figure can only be described, only when the position of the mirror line is well defined or known.

Under this type of transformation, the sizes of angles as well as the lengths of lines remain unchanged.

N/B y

x= -2 x= -1 x= 0 x=1 x=2



- The graph whose equation is x = 1, is a straight lines which is perpendicular to the x-axis, and passes through the point 1 on the x-axis.
- Also the line x = -2 passes through the point -2 on the x-axis.
- The y axis is also the same as the line x = 0

Types of reflections:

There are various types of reflections, and those to be considered are:

1. **Reflection in the y-axis or line x = 0:**

- For such a reflection,

$(x, y) \longrightarrow (-x, y)$.

Example (1).

P (2, 5) $\xrightarrow{\text{reflection in the y- axis}}$ P_1(-2,5).

Example (2)

If the Q(3, 8) undergoes a reflection in the line x = 0, then for its image

Q_1, $Q(3,8) \longrightarrow Q_1(-3,8)$.

2. **Reflection in the line y = b:**

For such a reflection, (x, y) $\longrightarrow$ (x, 2b – y).

Example (1)

If the point (2,4) undergoes a reflection in the line y = 3, then

(2, 4) $\xrightarrow{\text{reflection in line y = 3}}$ {2, 2(3) -4}

(2, 4) $\longrightarrow$ (2, 6 – 4)

(2, 4) $\longrightarrow$ (2, 2).

Example (2)

If the point Q(2,3) undergoes a reflection in the line y = 5, then for its image Q_1,

(x, y) $\xrightarrow{\text{reflection in the line y = b}}$ (x, 2b – y),

$\Rightarrow Q(2,3)\xrightarrow{reflection\ in\ line\ y = 5}$ Q_1{2, 2(5) -3}

Q(2, 3) $\longrightarrow$ Q_1(2,10 – 3)

Q(2, 3) $\longrightarrow$ Q_1(2,7).

N/B: In this case, (x, y) = (2, 3) and y = b becomes equal to y = 5.

$Therefore\ x = 2, y = 3\ and\ b = 5.$

There values are the substituted into the formula

$(x, y) \xrightarrow{\text{reflection in line } y = b} (x, 2b - y)$.

3. **Reflection in the x-axis or the line y = 0:**

- For such a reflection,

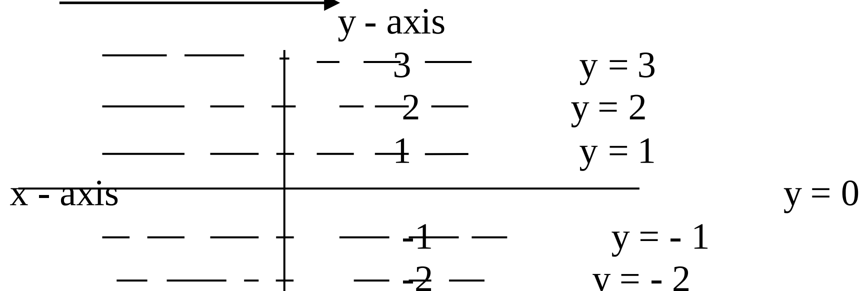
  $(x, y) \xrightarrow{\text{reflection in x-axis}} (x, -y)$

  Example (1)

  If P(4, 3) undergoes a reflection in the x-axis, then its image P_1 is given by $P(4,3) \xrightarrow{\text{reflection in x-axis}} P_1(4,-3)$.

  Example (2)

  If the point A(-3,-4) undergoes a reflection in the x-axis, then its image A_1, is given by $A(-3, -4) \xrightarrow{\text{reflection in a-axis}} A_1(-3, 4)$.



- The line graph whose equation is y = 3, is a straight line which is perpendicular to the y-axis, and passes through the point 3 on they-axis.
- Also the line y = -2, passes through the point -2 on the y - axis.
- Lastly the x- axis is the same as the line y = 0.

4. **Reflection in line y = x, or the line y-x = 0, or line –y = -x:**

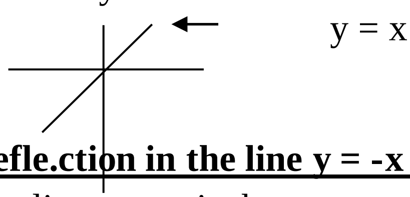
- The line y = x is the same as the line y-x = 0, or the line –y = - x
- For such a reflection,

  $(x,y) \xrightarrow{\text{reflection in line } y = x} (y, x)$.

  Example: if the point P(3, 5) undergoes a reflection in the line y = x, then for its image P_1,

  $P(3,5) \xrightarrow{\text{reflection in line } y = x} P_1(5, 3)$.

  The line y = x is shown next:



5. **Refle.ction in the line y = -x or the line y + x = 0 or the line – y = x:**

- The line y = -x is the same as the line y + x = 0, or the line -y = x.

- For such a reflection,

  $(x, y) \xrightarrow{\text{reflection in line } y = -x} (-y,-x)$.

  Example (1)

  If the point B(2, 5) undergoes a reflection in the line y = -x, then for its image B_1,

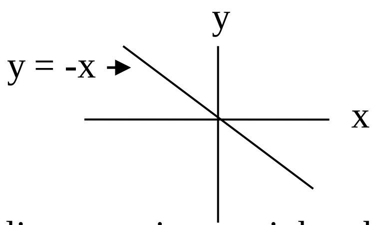
  $B(2,5) \xrightarrow{\text{reflection in line } y = -x} B_1(-5, -2)$.

  Example (2)

  If the point C(-3, -2) undergoes a reflection in the line y + x = 0, then for its image

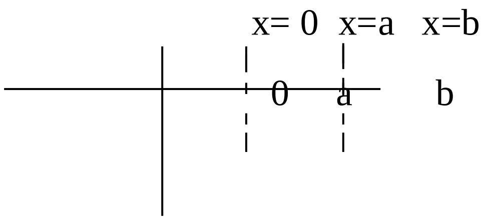
  $C(-3, -2) \xrightarrow{\text{reflection in line } y + x = 0} C_1(2, 3)$.

- Next is a diagrammatic representation of the line y = - x



N/B: The line x = a is a straight which is perpendicular to the x-axis, and passes through the point a, on the x-axis.

Also the line x = b is perpendicular to the x-axis, and passes through the point b on the x-axis.



6. **<u>Reflection in the line x = a:</u>**

   If the point (x, y) undergoes a reflection in the line x = a, then

   $(x, y) \xrightarrow{\text{reflection in line } x = a} (2a – x, y)$.

   Example (1)

   If the point (4, 3) undergoes a reflection in the line x = 5, then (x, y) = (4, 3) and x = a becomes equal to x = 5. Therefore x = 4, y = 3 and a = 5

   From $(x, y) \xrightarrow{\text{reflection in line } x = a} (2a-x, y)$.

   $(4, 3) \xrightarrow{\text{reflection in line } x = 5} \{2(5)-4, 3\}$.

   $(4, 3) \longrightarrow (10 – 4, 3)$

   $(4, 3) \longrightarrow (6, 3)$.

Example (2)

If the point p(-3, 4) undergoes a reflection in the line y = 8, then for its image P_1,

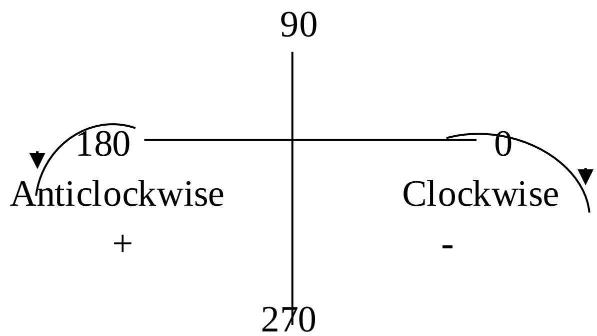
$P(-3, 4) \xrightarrow{\text{reflection in line } y = 8} P_1\{2(8) - (-3), 4\}$

$P(-3, 4) \longrightarrow P_1(16 + 3, 4)$.

$P(-3, 4) \longrightarrow P_1(19, 4)$.

Rotation :

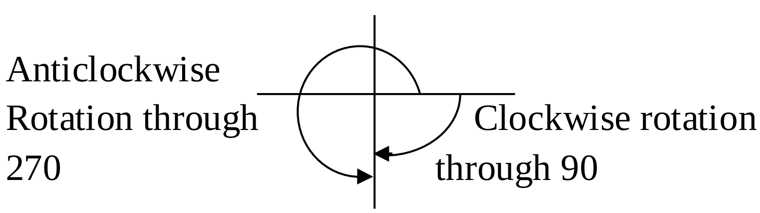
- This is measured in degrees and from the x-axis.
- It is either measured in a clockwise or an anticlockwise direction.
- Rotation in the clockwise direction is negative rotation, and that in the anticlockwise direction is positive rotation.



The different types of rotation to be considered are:

1. **<u>Clockwise rotation of 90 or rotation through -90°:</u>**

- This type of rotation is the same as an anticlockwise rotation through 270°or rotation through 270°.



From the sketch made, it can be seen that from the same starting point, a clockwise rotation through 90°, and an anticlockwise rotation through 270°, all meet on the same line or at the same point.

For this reason the two are the same. For a clockwise rotation through 90° or an anticlockwise rotation through 270°, about the origin

$(x, y) \longrightarrow (y, -x)$.

The following rotations are all the same, and as such the formula given must be used:

a) $(x, y) \xrightarrow{\text{Clockwise rotation through } 90° \text{ about the origin}} (y, -x)$.

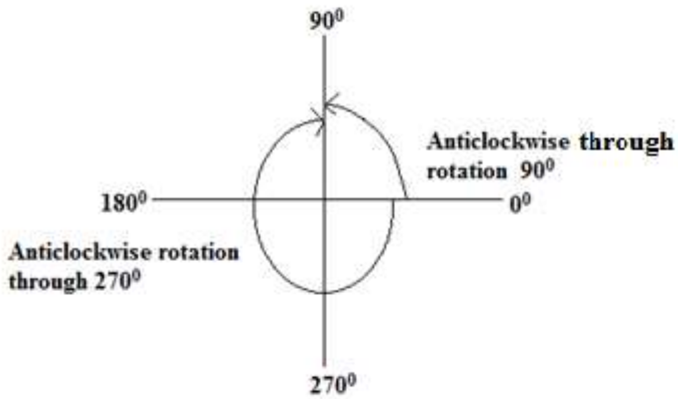
b) $(x, y) \xrightarrow{\text{Rotation through } -90° \text{ about the origin}} (y, -x)$.

c) (x, y) $\xrightarrow{\text{Anticlockwise rotation through } 270°}$ (y, -x).

d) (x, y) $\xrightarrow{\text{Rotation through } 270°}$ (y, x).

2. **Anticlockwise rotation through 90° or rotation 90°**

- This types of rotation is the same as clockwise rotation through 270° or rotation through -270°.



From the diagram drawn, it can be seen that an anticlockwise rotation through 90, and a clockwise rotation through 270, originating from the same starting point or line, all end at the same starting point or line.
For this reason, they are the same. If the point (x, y) undergoes an anticlockwise rotation through 90°, or clockwise rotation through 270°, then
(x, y) $\longrightarrow$ (-y, x).
The following transformations are the same, and questions based on them must be solved using the given formula:

a. (x, y) $\xrightarrow{\text{Anticlockwise rotation through } 90° \text{ about the origin}}$ (-y, x).

b. (x, y) $\xrightarrow{\text{rotation through } 90°}$ (-y, x)

c. (x, y) $\xrightarrow{\text{clockwise rotation through } 270°}$ (-y, x)

d. (x, y) $\xrightarrow{\text{rotation through } -270°}$ (-y, x)

Anticlockwise rotation through 90 or a clockwise rotation through 270 is known as or referred to as quarter turn.